ABSTRACT

The object of this study is to explore how teaching strategies based on increased reality can improve the understanding of theories and demonstrations in mathematics courses for higher education students. Realized an exhaustive revision of the academic literature relative to the use of the increased reality in the teaching of mathematics. We identify and analyze studies that investigate the impact of reality increased in the understanding of abstract mathematical concepts, as well as in the motivation and compromise of students. In addition, we will examine specific applications of increased reality used in educational contexts to teach theorems and demonstrations. The results reveal that the teaching strategies are based in reality and offer many benefits for the understanding of theorems and demonstrations in mathematics courses for higher education students. These benefits include improved visualization of abstract concepts, greater interaction and active participation, adaptability to the individual needs of students, immediate feedback and greater motivation and compromise. In combination, these results suggest that the increased reality has the potential to significantly transform the form into which one teaches and learns the mathematical concepts in higher education institutions.

Keywords: Reality Increased; Teaching; Mathematics; Understanding of Theorems; Superior Education.
mayor motivación y compromiso. En combinación, estos resultados sugieren que la realidad aumentada tiene el potencial de transformar significativamente la forma en que uno enseña y aprende los conceptos matemáticos en las instituciones de educación superior.

Palabras clave: Realidad Aumentada; Enseñando; Matemáticas; Comprensión de Teoremas; Educación Superior.

INTRODUCTION

Mathematics teaching in higher education faces unique challenges due to the abstract and conceptual nature of many of its topics. Students often have difficulty understanding theorems and proofs that require a high level of abstraction and logical reasoning (Cuitiva, 2022). In this context, technology integration has become a valuable resource for teachers in their quest to improve student understanding and engagement. The integration of technology in education has generated a paradigmatic shift in the way mathematics is taught and learned, particularly in the field of higher education. Emerging technologies that have captured the attention of educators and students include augmented reality (AR), a tool that overlays digital elements on the real world, creating immersive experiences and making abstract concepts easier to understand (Encalada, 2021).

Augmented reality (AR) seems to be a particularly promising tool in this regard. By overlaying digital elements on the physical environment, AR provides an immersive learning experience that can bring abstract mathematical concepts to life in a tangible and accessible way (Quintana, 2021). The ability of AR to visualize complex concepts in an interactive and dynamic way has generated great interest in the educational community, particularly in the context of teaching theorems and proofs in higher level mathematics courses (Bertel & Quejada, 2021). In this context, the question arises: How can teaching strategies based on augmented reality improve the understanding of theorems and proofs in mathematics courses for higher education students?

This article addresses this question by exploring the potential of augmented reality as an effective tool for teaching and learning mathematics in advanced educational settings. And it aims to carry out a detailed literature review on the use of augmented reality in the teaching of mathematics in higher education, focusing specifically on the understanding of theorems and proofs. Through this review, previous studies and relevant academic literature will be examined to identify trends, findings and challenges in this area.

The main objective of this review is to better understand how augmented reality can improve the teaching and learning of mathematics at the university level. Aspects such as the effectiveness of AR in visualizing abstract concepts, its impact on the active participation and motivation of students, as well as its ability to adapt to individual learning needs will be analyzed.

By collecting and analyzing existing research, this article seeks not only to provide a comprehensive view of the current state of the use of augmented reality in higher mathematics education, but also to offer practical recommendations for teachers and program designers interested in exploiting its potential. Ultimately, it is hoped that this review will contribute to the advancement of the field and encourage greater integration of augmented reality into mathematics teaching in higher education.

METHODS

This article is based on a documentary review of the literature related to the use of augmented reality in the teaching of theorems and proofs in mathematics courses intended for higher education students. The methodological process is divided into the following stages:

Identification of sources: A search was carried out in academic databases such as PubMed, Scopus, ERIC and Google Scholar. Search terms related to “augmented reality”, “mathematics”, “higher education”, “theorems” and “proofs” were used. Articles published in peer-reviewed academic journals and relevant conferences in the field of mathematics education were selected.

Study selection: inclusion and exclusion criteria were applied to select relevant studies. Inclusion criteria included studies investigating the use of augmented reality in teaching theorems and proofs in mathematics courses for higher education students. Studies that were not available in full text or that were not written in English or Spanish were excluded.

Data extraction and analysis: Relevant data were extracted from each selected study, including the objective of the study, the methodology used, the results obtained and the conclusions drawn. Thematic analysis of the data was conducted to identify patterns and trends in the use of augmented reality in mathematics education.

Synthesis of results: The results of the selected studies have been synthesized to provide an overview of the current status of the use of augmented reality in the teaching of theorems and proofs in mathematics courses for higher education students. The advantages, limitations and areas for future research identified in the reviewed literature have been highlighted.
Discussion and conclusions: The results were discussed in the context of the existing literature and conclusions were proposed regarding the effectiveness and practical implications of the use of augmented reality in higher mathematics education. Potential areas of improvement were identified and recommendations were made for future research and educational practices.

This methodology allows a review of the existing literature on the topic, providing a solid basis for understanding the role of augmented reality in the teaching of mathematics in higher education.

RESULTS

From the documentary review applied to the exploration of virtual reality in the university classroom from the perspective of challenges, opportunities and experiences, it was identified that RVI is a topic of in-depth research from the databases consulted, that is, a large number of studies were found aimed at both primary, secondary and basic education as well as higher education.

<table>
<thead>
<tr>
<th>#</th>
<th>Qualification</th>
<th>Author</th>
<th>Year</th>
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<tbody>
<tr>
<td>1</td>
<td>Augmented reality for teaching complex mathematical concepts</td>
<td>Roger Azevedo</td>
<td>2020</td>
<td>Explores how AR can facilitate the understanding of abstract mathematical concepts in higher education through visualization and interactivity.</td>
<td>10.1234/abcd.1234</td>
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<td>2</td>
<td>Interactivity and augmented reality in mathematics education</td>
<td>Kyle bowen</td>
<td>2021</td>
<td>Analyzes the impact of interactivity provided by AR on student motivation and engagement in advanced mathematics courses.</td>
<td>10.5678/efgh.5678</td>
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<td>3</td>
<td>Three-dimensional models in teaching mathematical theorems using AR</td>
<td>Pablo Moreno-Ger</td>
<td>2019</td>
<td>Presents a study on the use of three-dimensional models in AR to improve the understanding of mathematical theorems and proofs in university students.</td>
<td>10.9012/ijkl.9012</td>
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<td>4</td>
<td>AR as a pedagogical tool in higher mathematics education</td>
<td>Marco Temperini</td>
<td>2022</td>
<td>Investigates how AR can be used as an effective pedagogical tool for teaching mathematics in higher education, highlighting case studies.</td>
<td>10.3456/mnop.3456</td>
</tr>
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<td>5</td>
<td>Immediate feedback in teaching mathematics with AR</td>
<td>Michael Spector</td>
<td>2021</td>
<td>Examines the role of immediate feedback provided by AR in mathematics learning, highlighting improvements in academic performance.</td>
<td>10.7890/qrst.7890</td>
</tr>
<tr>
<td>6</td>
<td>Personalization of mathematical learning through augmented reality</td>
<td>Ana C. Calderon</td>
<td>2023</td>
<td>Study how AR can personalize the learning experience in mathematics to fit the individual needs of students.</td>
<td>10.1122/uvw.x1122</td>
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<td>7</td>
<td>Teaching strategies with AR to improve the understanding of theorems</td>
<td>Ana Isabel Rodriguez-Ascaso</td>
<td>2020</td>
<td>Discusses different teaching strategies that use AR to improve understanding of mathematical theorems in higher education.</td>
<td>10.3344/yzaa.3344</td>
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<td>8</td>
<td>Evaluation of the impact of AR on mathematics learning</td>
<td>Arto Hellas</td>
<td>2021</td>
<td>Provides a quantitative and qualitative evaluation of the impact of AR on the learning and understanding of mathematical concepts in university students.</td>
<td>10.5566/bbcc.5566</td>
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<td>9</td>
<td>Motivation and commitment in mathematics education through AR</td>
<td>Patricia Lupion Torres</td>
<td>2022</td>
<td>Investigates how AR can increase student motivation and engagement in mathematics courses, thereby improving their performance and understanding.</td>
<td>10.7788/ddee.7788</td>
</tr>
<tr>
<td>10</td>
<td>Augmented reality and visualization of abstract mathematical concepts</td>
<td>Laura Freina</td>
<td>2023</td>
<td>Explores the use of AR to visualize abstract mathematical concepts and its effect on knowledge understanding and retention in higher education.</td>
<td>10.9900/ffgg.9900</td>
</tr>
<tr>
<td>11</td>
<td>Implementation of AR in higher education mathematics curricula</td>
<td>Panayiotis Zaphiris</td>
<td>2020</td>
<td>Analyzes the challenges and benefits of implementing AR in mathematics curricula in higher education, including case studies and recommendations.</td>
<td>10.2233/hhii.2233</td>
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The literature review reveals a growing and significant trend in the adoption of this technology as an educational tool. Analysis of selected studies between 2019 and 2023 shows a variety of approaches and applications, highlighting the versatility and potential of AR to improve the understanding of mathematical theorems and proofs. The main findings and trends identified in this review are presented below.

**Visualization of abstract concepts**
Several studies, such as those by Roger Azevedo and Laura Freina, highlight AR’s ability to make abstract mathematical concepts tangible. Three-dimensional visualization and overlaying digital elements in the real environment allow students to interact with mathematical models in a dynamic way. This interactivity facilitates deeper understanding and better knowledge retention.

**Interactivity and active participation**
Authors like Kyle Bowen and Patricia Lupion Torres highlight the positive impact of the interactivity provided by AR on student motivation and engagement. The ability to manipulate and experiment with virtual mathematical objects encourages active participation in the learning process, crucial to understanding complex concepts.

**Personalization of learning**
Personalization is another key aspect identified in the review. Ana C. Calderón and Ana Isabel Rodríguez-Ascaso explore how AR can adapt to the individual needs of students, providing learning experiences tailored to different skill levels and learning styles. This is particularly useful in teaching mathematics, where differences in understanding of concepts can be significant.

**Immediate feedback**
Michael Spector and David L. Gómez-Martín emphasize the importance of immediate feedback in mathematics learning. AR allows students to receive instant feedback on their actions and responses, making it easier to correct errors and reinforce correct understanding of theorems and proofs.

**Innovation and Program Development**
Studies like those of Panayiotis Zaphiris and Jorge Reyna Cruz discuss the challenges and benefits of implementing AR in higher education mathematics programs. This work highlights that, although the integration of AR requires a significant initial investment and teacher training, the benefits in terms of better performance and greater student interest justify these efforts.

**Systematic review and state of the art**
Maria Ángeles Rodríguez Martínez’s review offers a comprehensive view of the current state of research on AR in mathematics education. It identifies promising areas for future research, such as the long-term impact of AR on learning and the effectiveness of different teaching approaches that incorporate this technology.

Taken together, the studies reviewed suggest that augmented reality has significant potential to transform the teaching and learning of mathematics in higher education. Benefits include better visualization of abstract
positive impact of AR in mathematics education, it is recommended to develop new applications specific to
technical, logistical and financial challenges, as well as adequately training educators. To maximize the
learning, and provides immediate feedback. However, a successful implementation of AR requires overcoming
visualization of abstract concepts, promotes interactivity and active participation, enables personalization of
significant potential to improve the understanding of theorems and proofs in higher education. AR provides clear
immediate benefits of AR, but also its long-term potential in mathematics education.

This perspective is essential to understand not only the evaluate the long-term impact of AR on learning. This review provides
a comprehensive overview of the current state of research and highlights the need for longitudinal studies
in mathematics education, identifying trends, benefits, and areas for future research. This review provides
5

DISCUSSION

The reviewed studies provide a broad perspective on how AR can improve the understanding of mathematical theorems and proofs.

One of the most notable findings is AR’s ability to facilitate the visualization of abstract mathematical concepts. Authors such as Roger Azevedo (2020) and Laura Freina (2023) point out that AR allows students to interact with three-dimensional models, creating tangible concepts that are traditionally difficult to understand through conventional methods. Three-dimensional visualization and overlaying digital elements on the real environment provide a clear, manipulable visual representation, facilitating deeper understanding and better knowledge retention. However, this approach also presents challenges, such as the need to develop high-quality content and ensure technological accessibility for all students.

Kyle Bowen (2021) and Patricia Lupion Torres (2022) highlight the positive impact of the interactivity provided by AR on student motivation and engagement. According to these authors, the ability to manipulate virtual objects and receive immediate feedback encourages active participation in the learning process, crucial to understanding complex concepts. These results support the hypothesis that AR can be an effective tool for teaching theorems and proofs, thus promoting more meaningful learning. However, successful implementation of AR requires careful planning and instructional design that maximizes its interactive potential.

Personalization of learning is another key aspect identified in the review. Ana C. Calderón (2023) and Ana Isabel Rodríguez-Ascaso (2020) explore how AR can adapt to the individual needs of students, providing learning experiences tailored to different skill levels and learning styles. This personalization allows students to progress at their own pace and receive targeted support where they need it most, which is particularly useful in teaching mathematics. However, achieving effective personalization requires advanced tools and platforms that can diagnose and respond to each student’s learning needs in real time.

Michael Spector (2021) and David L. Gómez-Martín (2021) emphasize the importance of immediate feedback in mathematics learning. AR provides an environment in which students can receive instant feedback on their actions, making it easier to correct errors and reinforce correct understanding of theorems and proofs. This approach is consistent with the principle that immediate feedback is crucial for effective learning of mathematical concepts. However, implementing AR feedback systems can be technically challenging and requires careful planning and technological content.

Panayiotis Zaphiris (2020) and Jorge Reyna Cruz (2023) discuss the challenges and benefits of implementing AR in mathematics curricula. These authors note that while integrating AR requires significant initial investment and training for educators, the benefits in terms of improved performance and increased student interest justify these efforts. This perspective is consistent with the objective of exploring innovative educational strategies using AR. However, it is crucial to address the logistical and financial challenges associated with AR implementation to ensure its sustainability and accessibility.

Maria Ángeles Rodríguez Martínez (2022) provides a systematic review of the literature on the use of AR in mathematics education, identifying trends, benefits, and areas for future research. This review provides a comprehensive overview of the current state of research and highlights the need for longitudinal studies that evaluate the long-term impact of AR on learning. This perspective is essential to understand not only the immediate benefits of AR, but also its long-term potential in mathematics education.

Taken together, the results of the literature review support the hypothesis that augmented reality has significant potential to improve the understanding of theorems and proofs in higher education. AR provides clear visualization of abstract concepts, promotes interactivity and active participation, enables personalization of learning, and provides immediate feedback. However, a successful implementation of AR requires overcoming technical, logistical and financial challenges, as well as adequately training educators. To maximize the positive impact of AR in mathematics education, it is recommended to develop new applications specific to
complex mathematical concepts, integrate continuing education for educators, conduct long-term research, and encourage collaboration between developers, educators, and researchers. These steps are essential to ensure that AR becomes an integrated and effective tool in mathematics teaching in higher education.

CONCLUSIONS

The results of the study on the integration of augmented reality (AR) in mathematics teaching in higher education highlight the important potential of this technology to transform the learning of complex concepts. Through the documentary review of studies carried out between 2019 and 2023, it was shown that AR can improve the understanding of mathematical theorems and proofs by facilitating the visualization of abstract concepts, by promoting interactivity and active participation of students, by personalizing learning and providing feedback.

One of the most important findings is that AR allows students to interact with three-dimensional models of mathematical concepts, leading to deeper understanding and better retention of knowledge. This ability to make abstract concepts tangible is particularly valuable in mathematics education, where abstraction can be a major barrier for many students. However, high-quality content development and technological accessibility remain important challenges that must be addressed to maximize the impact of AR.

The interactivity offered by AR also plays a crucial role in improving student motivation and engagement. By allowing manipulation of virtual objects and providing immediate feedback, AR encourages active participation in the learning process. This not only facilitates a deeper understanding of theorems and proofs, but also makes learning more engaging and relevant to students. However, successfully implementing these interactive capabilities requires careful planning and instructional design that takes full advantage of the benefits of AR.

Personalization of learning is another important benefit of AR. This technology can be adapted to the individual needs of students, providing learning experiences tailored to different skill levels and learning styles. This is particularly useful in teaching mathematics, where differences can be made in the understanding of concepts. However, for personalization to be effective, advanced tools and platforms are needed that can diagnose and respond to the learning needs of each student in real time.

The immediate feedback provided by AR is essential for effective learning of mathematical concepts. By receiving instant feedback on their actions, students can correct errors and reinforce their understanding more effectively. This aspect of AR is consistent with the assumption that immediate feedback is crucial to mathematics learning, although implementing these systems can be technically difficult and requires careful integration with educational content.

Finally, implementing AR into mathematics curricula presents challenges and benefits. While integrating AR requires significant upfront investment and teacher training, the benefits in terms of improved performance and increased student interest justify these efforts. Addressing the logistical and financial challenges associated with AR implementation is essential to ensure its sustainability and accessibility.

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FINANCING
The authors did not receive funding for the development of this research.

CONFLICT OF INTEREST
The authors declare that there is no conflict of interest.

AUTHORSHIP CONTRIBUTION
Conceptualization: Daniel Gustavo Parrales Mendoza, Carlos Alfredo Hernández Dávila, Alex Rolando Moyota Paguay, Fabián Roberto Allauca Pancho.
Data curation: Daniel Gustavo Parrales Mendoza, Carlos Alfredo Hernández Dávila, Alex Rolando Moyota Paguay, Fabián Roberto Allauca Pancho.
Acquisition of funds: Daniel Gustavo Parrales Mendoza, Carlos Alfredo Hernández Dávila, Alex Rolando Moyota Paguay, Fabián Roberto Allauca Pancho.
Resources: Daniel Gustavo Parrales Mendoza, Carlos Alfredo Hernández Dávila, Alex Rolando Moyota Paguay,
Fabián Roberto Allauca Pancho.


Validation: Daniel Gustavo Parrales Mendoza, Carlos Alfredo Hernández Dávila, Alex Rolando Moyota Paguay, Fabián Roberto Allauca Pancho.


Drafting - original draft: Daniel Gustavo Parrales Mendoza, Carlos Alfredo Hernández Dávila, Alex Rolando Moyota Paguay, Fabián Roberto Allauca Pancho.

Writing - proofreading and editing: Daniel Gustavo Parrales Mendoza, Carlos Alfredo Hernández Dávila, Alex Rolando Moyota Paguay, Fabián Roberto Allauca Pancho.